REPRESENTATION OF THE GLOBAL GRAVITY FIELD BY POINT MASSES ON OPTIMIZED POSITIONS BASED ON RECENT SPHERICAL HARMONICS EXPANSIONS

F. Barthelmes, R. Dietrich and R. Lehmann

Central Institute for Physics
of the Earth,
Telegrafenberg A17
O-1561 Potsdam
Germany

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1. Motivation and Background

The principal possibility to represent the outer gravitational potential of the earth (or of an other planet) by superposition of potentials of point masses (where the masses are distributed in the earth's interior) is evident from discretisation of Newtons representation (1) and is surely wellknown for a long time.

$$U(P) = G \iiint_V \varrho(q)\, l^{-1}(P,q)\, dv(q) = \lim_{\substack{\Delta v_i \to \infty \\ N \to \infty}} G \sum_{i=1}^{N} \varrho(q_i)\; v_i\, l^{-1}(P,q_i) \qquad (1)$$

(P - outer point; q - inner point; q_i - point within the volume element v_i
$l(P,q)$ - distance between P and q ; ϱ - mass density)

The idea of using point masses in geodesy is ascribed to J.A.Weightman who surely has stimulated the following investigations in this field decisively by his paper in Athen (Weightman, 1965). The publications of the soviet colleagues went a little bit unheeded (because of the language barrier): Kupradze and Aleksidze (1964) proved that point mass potentials are linearly independent and complete if the masses are located on a closed surface inside the earth, and Aleksidze discussed in (Aleksidze, 1966) the possibility to approximate the disturbance potential by point masses.
It is interesting that in the first application the gravitational field of the moon was approximated and not that of the earth (Muller, Sjogren, 1968). The reason for that are on the one hand the structure of the moons field which really is characterized by the so called mascons and on the other hand by the fact that, because of the observability of the lunar probes, there was much more gravitational field information for the front than for the back of the moon and one had to choose an adequate representation.
Among the great number of publications which followed thereupon the ones by Needham (1970) and Balmino (1972, 1974) should be mentioned here. The basic idea was to distribute the point masses regularly below the desired area and to determine the magnitudes of the masses by least squares approximation of the measurements (functionals of the gravitational field). At that time the most important motivations to use the point mass method were:
- adaptability to spatial distribution of measurements, i.e. the possibility of increasing the number of parameters without perturbing distant regions
- discrete formulation leading to simple programming
- possibility of simple combination of observations from different sources
- possible connections to geophysics

Balmino (1974) raised the question, whether the number of the needed point masses can be diminished substantially, if not only the magnitudes but also the positions of the point masses are optimized. The approximation of the outer gravitational field by point masses with optimizing the masses and position of the point masses leads to a complicated non-linear optimization problem. In connection with the solution of this problem a number of questions of approximation theory as well as questions related to the inverse problem of potential theory have to be solved. A detailed discussion of these problems and the formulation and theoretical foundation of a corresponding approximation algorithm one can find in (Barthelmes, 1986). Practical results, globally as well as locally, based on this algorithm were presented in (Dietrich and Gendt, 1989), (Dietrich, Gendt, Barthelmes, 1990), (Barthelmes, 1988), (Barthelmes and Dietrich, 1990).

During the last years some aspects of motivation to use point mass methods for gravity field approximation have been changed. With the availability of faster and faster computers having more and more storage (and with the possibility to grid and to interpolate any representation) the problem to find a mathematical representation being very fast in operational work seems to play not any longer a dominant role (nevertheless this aspect will not be completely meaningless for special aims). Also the adaptability of some mathematical representation to irregular distributed data probably will lose its importance in future if satellite gradiometry, besides altimetry, will provide dense gravity information over the whole earth. But surely one will not renounce the concept of overlaying global, regional, and local models (with higher and higher resolutions) - and it should be advantageously to use a mathematical representation which is suitable for it in like manner (e.g. the point mass method) because it enables an unique mathematical modelling.

On the other hand, in our opinion the point masses' connection with geophysics (i.e. the possibility of modelling the outer gravitational field as sum over the fields of spherically symmetric density anomalies inside the earth) becomes more important. Besides the more and more detailed knowledge about the outer gravitational field the data describing the inner structure of the earth (esp. the density distribution) will obviously become better too because of more and more complex geological-geophysical treatment of the problem. As great aim one can formulate (even if an acceptable solution is in the distant future):

Permanent improvement of a model, which is in full agreement with all data concerning the inner structure (density, temperature, pressure, caloric conductivity, velocity of seismic waves etc.) as well as with all data concerning the outer gravitational- and magnetic field.

How could such a model be parameterized ?.

Point masses (-potentials) are on the one hand, from mathematical point of view, the fundamental solutions of Laplace's equation and on the other hand, from physical view point, the simplest sources of gravity. All possible outer potentials as well as all possible density structures can be approximated arbitrarily accurate by point masses (e.g. spherically symmetric density anomalies). Although enough information about the earth interior will not be available in the near future to tackle the above formulated aim seriously and the combination of potential data and earth-interior-data will require immense research work, it seems reasonable to test the point mass method with mass-position-optimizing concerning its efficiency in geodesy.

In the following the algorithm deduced in (Barthelmes,1986) (and published in detail up to now in German only) will be presented once more including some theoretical aspects followed by practical results based on the geopotential coefficient gravity model OSU89B (complete to degree 360) (Rapp and Pavlis, 1990).

The basic idea of the approximation algorithm consists in approximating a set of given boundary values (in our case gradient vectors of the disturbance potential) by increasing the number of point masses step by step up to a desired accuracy. The adjusted parameters are the magnitudes and the positions of the point masses and the number of masses to reach a defined accuracy should be as small as possible.

2.Theoretical Formulation of the Algorithm

2.1.Some Theoretical Presumptions

Let Ω be the open set in E^3 outside a sphere σ with center at the origin. Consider the space H of all functionals U for which holds :

- U is two times continuously differentiable in Ω :

$$U \in C^{(2)}(\Omega) \tag{2}$$

- U satisfies Laplace's equation in Ω :

$$\Delta U = 0 \tag{3}$$

- U is regular at infinity:

$$|U(P)| = O(1/|P|) \; ; \quad |\operatorname{grad} U(P)| = O(1/|P|^2) \text{ if } |P| \to \infty \tag{4}$$

$$(P \in \Omega \; ; \; O(*) = \text{Landau-Symbol})$$

The space H of harmonic functions may be equipped with the wellknown inner product

$$(U|V) = 1/4\pi \int_\sigma U\, V\, d\sigma \tag{5}$$

or

$$(U|V) = 1/4\pi \int_\sigma \operatorname{grad}U \operatorname{grad}V\, d\sigma \tag{6}$$

and if we only require functions having square integrable boundary values H becomes a Hilbert space.
The set of point mass potentials $\{ \phi_i(P) \}$

$$\phi_i(P) = |q_i - P|^{-1} \quad ; \; q_i \in E^3 \backslash \bar{\Omega} \; ; \; P \in \Omega \tag{7}$$

is complete in H, if $\{q_i\}$ (i.e. the point mass positions) is a dense point sequence on σ (or on another regular inner surface) (Kupradze, Aleksidze, 1964), (Freeden, 1980). That means any potential can be approximated arbitrarily accurate by point mass potentials.
Any countable set of point mass potentials are linearly independent, if the masses are arbitrarily distributed in the inner region provided that $q_i \neq q_j$ if $i \neq j$ (Stromeyer, Ballani, 1984). That means, two configurations of a finite number of point masses inside the earth produce in any case different potentials, if both point mass configurations are not identical.
With that the theoretical presumptions are given to optimize in practical gravity field approximations by point masses also the positions of the masses. In this connection it is worth mentioning the fact, that the inverse gravimetric problem (which is nonunique for continuous density distributions) becomes unique if the solution is restricted to a countable number of point masses (density δ-pulses).

2.2. The Approximation Algorithm

The anomalous potential $T \epsilon H$ should be approximated. Although the point mass potentials are not orthogonal let $\{ h_i \epsilon H : (i=1, 2, 3,...)\}$ be an orthonormal basis. From the set $\{h_i\}$ N vectors $\{h_{i1}, h_{i2}, ..., h_{iN}\}$ should be selected such, that (for fixed N) the vector T will be approximated as good as possible:

$$\| T - \sum_{m=1}^{N}(T | h_{im}) \| = \min \left\{ \| T - \sum_{m=1}^{N}(T | h_{km})h_{km} \| \ : k_m \epsilon N_Z \right\} \tag{8}$$

(N_Z - set of natural numbers)

It is easy to show, that the vectors $\{h_{im}\}$ can be selected by looking for that inner product with the vector T which has the greatest absolute value. For the N-th step of stepwise approximation that means :

$$(T | h_{iN})^2 = \max \left\{(T - \sum_{m=1}^{N-1} (T | h_{im})h_{im} | h_{kN})^2 : k_N \epsilon N_Z\right\} \tag{9}$$

Let us now consider the approximation using as base functions the non-orthogonal point mass potentials. Because of the non-linearity if a new point mass is added, strictly speaking, all previous point mass positions have to be changed (by non-linear optimization) again. On principle it is no problem to find the nearest local minimum for given start values by the help of the known methods of non-linear optimization. Consequently the problem consists in finding reasonable values for the point mass positions. The idea is, to use at each step (adding of a new point mass) the previous point mass positions as approximate values for the optimization procedure. The start position for the new mass can be found like in the orthogonal case (9).

Now the algorithm can be formulated mathematically:

Notation: $\Phi_i = \phi_i \| \phi_i \|^{-1}$ (normed base function to have $\| \Phi_i \| = 1$)

μ_i - normed coefficient

additional upper indices (e.g. Φ_i^N , q_i^N) to take into consideration the fact, that all point mass positions are different for different approximation levels N

$\Phi_i^k = \Phi(P,q_i^k)$; $\Phi^* = \Phi(P,q^*)$

E^n - n-dimensional euclidian space

1-st step

Determination of q_1^1 (i.e. Φ_1^1) and μ_1^1 by

$$(T|\Phi_1^1)^2 = \max\left\{(T|\Phi^*)^2 : q^* \in E3\backslash\bar{\Omega}\right\} \quad \text{and } \mu_1^1 = (T|\Phi_1^1)$$

N-th step

a) Determination of the approximate values $\approx{q}_i^N$ (i.e. $\approx{\Phi}_i^N$) $(i=1,\ldots,N-1)$ by

$\approx{q}_i^N = q_i^{N-1}$ (i.e. $\approx{\Phi}_i^N = \Phi_i^{N-1}$)

b) Determination of the approximate value $\approx{q}_N^N$ (i.e. $\approx{\Phi}_N^N$) by

$$\left(T - \sum_{i=1}^{N-1} \mu_i^{N-1}\, \Phi_i^{N-1} \,\middle|\, \approx{\Phi}_i^{N-1}\right)^2 = \max\left\{\left(T - \sum_{i=1}^{N-1} \mu_i^{N-1}\, \Phi_i^{N-1} \,\middle|\, \Phi^*\right)^2 : q^* \in E^3\backslash\bar{\Omega}\right\}$$

c) Determination of q_i^N (i.e. Φ_i^N) and μ_i^N $(i=1,\ldots,N)$ by

$$\left\| T-\sum_{i=1}^{N}\mu_i^N\, \Phi_i^N \right\| = \min\left\{ \left\| T-\sum_{i=1}^{N}\mu_i^*\, \Phi_i^* \right\| : q_i^* \in E3\backslash\bar{\Omega}\ ;\ \mu_i^* \in \bar{E}\ ,\ (i=1,\ldots,N) \right\}$$

The described procedure is justified especially if the used point mass potentials are "nearly orthogonal", which is better fulfilled for the inner product (6) instead of (5).

3. Practical Realization of the Algorithm

Boundary values:
acceleration vectors equally distributed over the sphere σ

Determination of the start position of each new point mass:
Since only one base function (point mass position) has to be searched for the principle of equ.(9) can be used which is equivalent to the minimization :

$$\| T_{N-1} - \mu_N^N\, \Phi_N^N \| = \min\left\{ \| T_{N-1} - \mu^*\, \Phi^* \| : q_i^* \in E3\backslash\bar{\Omega}\ ;\ \mu_i^* \in E \right\} \quad (10)$$

$$\text{with} \quad T_{N-1} = T - \sum_{i=1}^{N}\mu_i^N\, \Phi_i^N$$

Therefore it is only necessary to find a start position for the non-linear optimization (10), which on its part gives the start position for the N-th point mass. This start position for (10) one can find by putting the mass below the maximum value of $|\text{grad}\, T_{N-1}(P)|$, $P \in \sigma$.

Additional simplification at the practical realization:

Not all point mass positions are improved at each step - all those point masses the scalar product of which with the new point mass is smaller than a given value S_{eps} remain unchanged.

For each approximation level it is possible to make an additional (linear) adjustment of the magnitudes for fixed point mass positions using several properly, mutually weighted functionals of the gravity field.

4. Some Aspects of Nonlinear Least Squares Estimation

The point mass models with optimized positions are obviously nonlinear. Therefore many wellknown properties of parameter estimation in linear models can not be applied. Generally we have for example

$$E\{f(x)\}=f(E\{x\}) \qquad (11)$$

Simple propagation laws for covariances or for whole distributions as in the linear case do not hold rigorously.
We assume gauss-distributed observations **d** with weight matrix **P**, we estimate parameters **x** (magnitudes and positions of point masses) by means of the least squares method and finally we predict gravity field functionals **f**.

Model: $E\{\mathbf{d}\}=\mathbf{G}(\mathbf{x}), \quad \mathrm{cov}\{\mathbf{d}\}=\sigma_0^2\mathbf{P}^{-1}$

Est. principle: $[\mathbf{d}-\mathbf{G}(\hat{\mathbf{x}})]\mathbf{P}[\mathbf{d}-\mathbf{G}(\hat{\mathbf{x}})]=\mathrm{min.}$

Estimate: $\hat{\mathbf{x}}=\mathbf{G}^-(\mathbf{d})$ ($\mathbf{G}^-$ is the least squares estimator)

Prediction: $\hat{\mathbf{f}}=\mathbf{F}(\hat{\mathbf{x}})$

At least as important as the estimates themselves are of course values to describe, how much one can rely on them. This could be the variances, but one should have in mind, that estimators in nonlinear models are not completely determined by mean and variance. Premature conclusions could be misleading.
Besides there are some obstacles even to get realistic (say unbiased) means and variances:

1st bias source: Nonlinearity

Generally the bias in the mean

$$E\{\mathbf{x}-\hat{\mathbf{x}}\}=\mathbf{G}^-(E\{\mathbf{d}\})-E\{\mathbf{G}^-(\mathbf{d})\}$$

is indeed nonzero (cf.(11)). The same holds true for **f**. To obtain variances we linearize $\mathbf{G}^-$,**F** and apply the linear covariance propagation laws:

$$\mathrm{cov}\{\hat{\mathbf{x}}\} \approx (\delta\mathbf{G}^-/\delta\mathbf{d})\big|_{\mathbf{d}}\, \mathrm{cov}\{\hat{\mathbf{d}}\}\, (\delta\mathbf{G}^-/\delta\mathbf{d})\big|_{\mathbf{d}} \qquad (12)$$

$$\mathrm{cov}\{\hat{\mathbf{f}}\} \approx (\delta\mathbf{F}/\delta\mathbf{x})\big|_{\hat{\mathbf{x}}}\, \mathrm{cov}\{\hat{\mathbf{x}}\}\, (\delta\mathbf{F}/\delta\mathbf{x})\big|_{\hat{\mathbf{x}}}$$

Terms including higher derivatives of $\mathbf{G}^{-}$ or $\mathbf{F}$ are truncated. Therefore the obtained values are again incorrect. The amounts of bias due to nonlinearity depend on the noise level and on model design. So we should investigate, how much noise could be permitted and which model design minimizes the bias.

2nd bias source: Incomplete optimization

In order to save computation time we should not go too far in our stepwise nonlinear optimization procedure, symbolically represented by

$$x_{n+1}=x_n+\delta x_n \quad , \text{ where } n=0,1,...$$

But our feeling of convergence is rather deceivable and we have no simple means to assess the bias $|\mathbf{x}_\infty-\mathbf{x}_n|$ or $|\mathbf{F}(\mathbf{x}_\infty)-\mathbf{F}(\mathbf{x}_n)|$. Moreover, the solution $\hat{\mathbf{x}}$ should depend on nothing else than $\mathbf{d}$. Virtuelly estimates are fully undetermined, if they depend on fully uncertain quantities like x_o occasionally. But our choice of x_o is based on detailed investigations and it is to some extend certain. Further studies should indicate, how serious this bias really could be.

3rd bias source: Inexact theory

Everywhere in gravity field modelling we are faced with more or less arbitrary choices (norms, base functions etc.). Some features of the field remain unmodelled and cause a systematic deviation between estimated and true values. So one should not expect to get unbiased estimates even in linear models. We have imposed the conditional probability

$$P(\mathbf{x}|\mathbf{d})=\delta(\mathbf{G}(\mathbf{x})-\mathbf{d}) \qquad (\delta \text{ denotes DIRAC's delta function})$$

Virtuelly a continuous distribution seems more realistic, but is not available. "Self-contructed" $P(\mathbf{x}|\mathbf{d})$ could be even more inexact.

In most cases the inexact theory of our model generates probably more bias than nonlinearity or incomplete optimization as our experiences so far indicate. But many questions remain open for further studies, where we intend to rely on Monte-Carlo-methods.
In this investigation we estimate variances by treating the mass positions as fixed. So we avoid nonlinearity and optimization problems and we save a great deal of computation time. Nevertheless, this method is more inexact by far, generally it leads to very optimistical error estimates. By comparing some "true" and estimated prediction errors we will see, whether this could be done or not.

How many point masses are necessary ?

Every additional point mass of course improves the fit of the data (sum of squares of residuals), but at the same time decreases the number of degrees of freedom. With very few point masses their ratio σ_o^2 is rapidly decreasing, however there is a number of point masses, where σ_o is minimized. This could serve as a termination criteria for the approximation procedure. Closely examined this gives always some point masses more than necessary, as hypothesis testing (F-test) indicates. Practically the latter is difficult to apply, because some conditions will very seldom or never be satisfied. So it seems reasonable to trust in some prior information about σ_0, which is available in most cases (e.g. from repeated measurements). Consequentely we can terminate, if our estimated noise level in the data coincides with the a priori given one.

5.Numerical Results

The numerical results are mainly based on input data generated from the model OSU89B. They are presented at the right side of the poster, in detail:

5.1. The Normal Field GRS80 (7PM)

5.2. Global Model of the Long Wavelength Part (PM107/01)

5.3. Global Model with Medium Resolution (PM1107/01)

5.4. Regional Model for Europe (PM2107/01)

5.5. Global Model with High Resolution

We have usually applied an overlay principle for the models, that means: a model with higher resolution or a regional model is based on a model with lower resolution etc.

6.Conclusions

- The point mass positions are following the structure of the gravity field (e.g. higher number of masses in rough areas - high data density presupposed) and the procedure can be stopped at desired approximation accuracy.

- Models with different resolutions (ellipsoid, long wavelength, short wavelength) and for various purposes can be handled and combined very effectively with the same mathematical procedure.

- The regional investigations for Europe show, that the model with 2107 point masses could be used as a reference field alternatively to OSU89B. The OSU89B-Geoid in Europe was approximated with dm-accuracy. These investigations are therefore hopeful prerequisits for further geoid research in Europe based on the point mass method, including the real data.

- Questions of error estimation in point mass models need further research.

- For the Europe window (about 5% of the Earth surface) 1000 additional point masses were needed to reach dm-accuracy for the geoid. Therefore one can expect (as a raw estimate), that a global model of about 20 000 point masses satisfies this accuracy requirement.

- As it is visible in the global model with high resolution the point mass positions can be related to geophysical structures. Including additional informations (e.g. topography/bathymetry, density etc.) point masses can hopefully be a tool for geophysical interpretations.

5.1. Approximation of the Normal Field GRS 80

(7 Point Masses)

The point masses are situated following a proposal of HEIKINNEN (1980):

- **on the mean rotation axis of the earth (z-axis)**
- **equidistantly**
- **equatorial-symmetrically**

Their magnitudes fit the level ellipsoid of GRS 80 with a

mean accuracy of 2.0 mm

and a

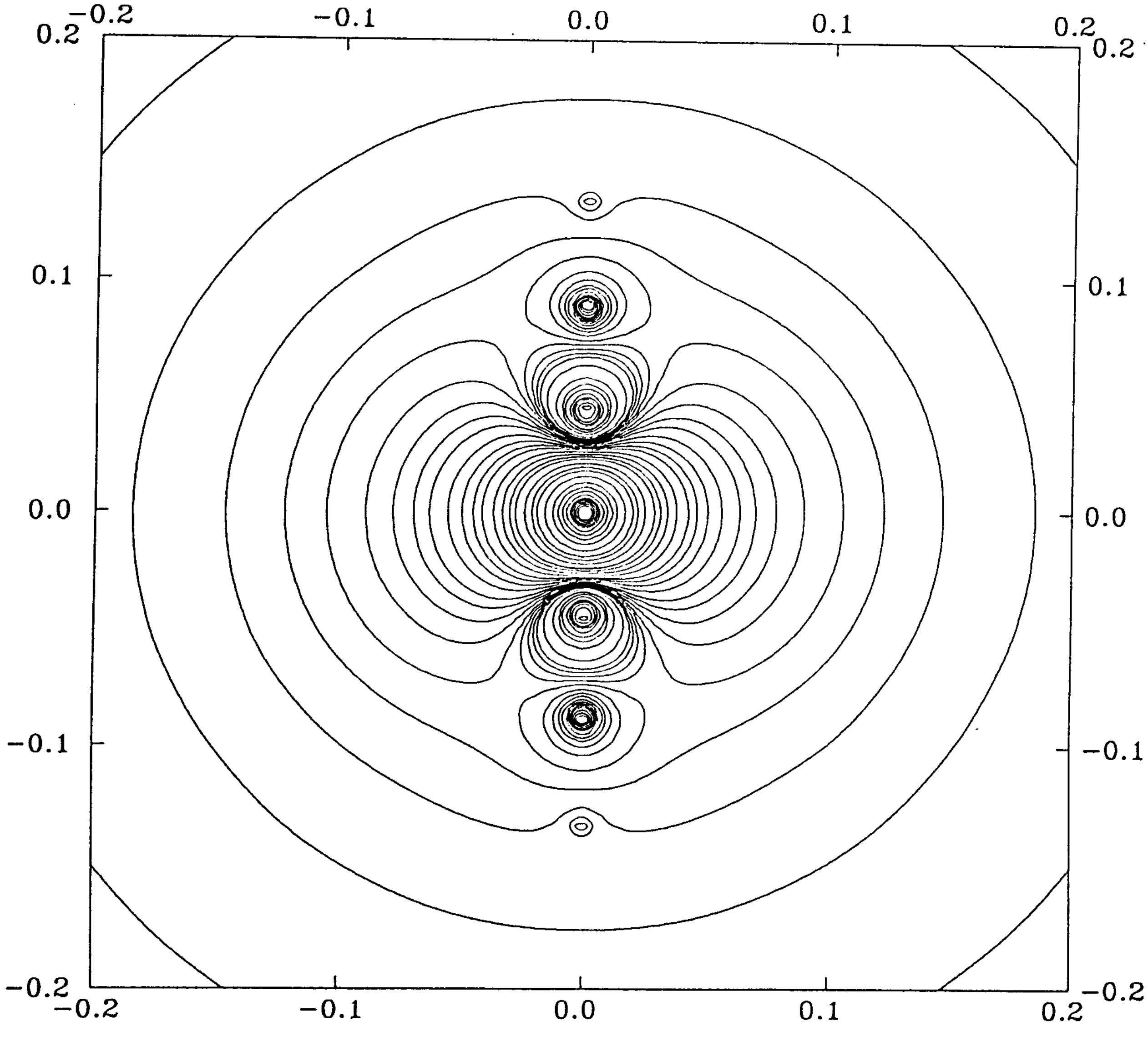
maximum deviation of 2.7 mm.

An even better approximation could be obtained (smaller distances and stronger magnitudes), but causes unacceptable rounding errors during evaluation.

GRS 80 approximation (positions on the z-axis in units of R_E, magnitudes in units of GM):

Z	magnitude
- 0.132	- 0.0086592
- 0.088	0.1016124
- 0.044	- 0.6081218
0.000	2.0303372
0.044	- 0.6081218
0.088	0.1016124
0.132	- 0.0086592

Visualization of the potential GRS 80
- meridional cross-section
- equipotential lines
- length scale: units of earth radius



5.2. Global Model of the Long-Wavelength-Part

(7 + 100 Point Masses): PM107/01

Aim: e.g. orbit calculations of high altitude satellites like LAGEOS and GPS

Input Data: OSU 89B up to (20,20) minus GRS 80 (7 Point Masses)

- for position optimization:
  gridded acceleration vectors on a reference sphere

- for final adjustment of masses (positions fixed):
  spherical harmonic coefficients, weighted with respect to their variance and their influence at 3000 km altitude

R.M.S. Values of Acceleration Differences for Several Gravity Field Models in LAGEOS Altitude (in 10^{-8} m · sec^{-2} = μgal)

	GRIM 4	TEG-2	GEM-T2	OSU89B
TEG-2	1.7			
GEM-T2	1.2	1.5		
OSU89B	1.3	1.4	0.8	
PM107/01	1.3	1.5	0.9	0.4

for comparison: r.m.s. of acceleration in LAGEOS altitude (without normal field GRS 80): 935 · 10^{-8} m · sec^{-2}

GRIM 4: Deutsches Geodätisches Forschungsinstitut, Abt. I, München/Germany
Groupe de Recherches de Geodesie Spatiales CNES, Toulouse, France

TEG-2: University of Texas, Austin/USA

GEM-T2: Goddard Space Flight Center, Greenbelt/USA

OSU89B: Ohio State University, Columbus/USA

PM107/01: this investigation

Test computation

Orbital fit for satellite LAGEOS:

30-day-arc (MJD: 47 946,5 - 47 976,5)
orbital program system POTSDAM 5

OSU89B: $\pm$ 6,2 cm
PM107/01: $\pm$ 6,9 cm

5.3. Global Model with Medium Resolution

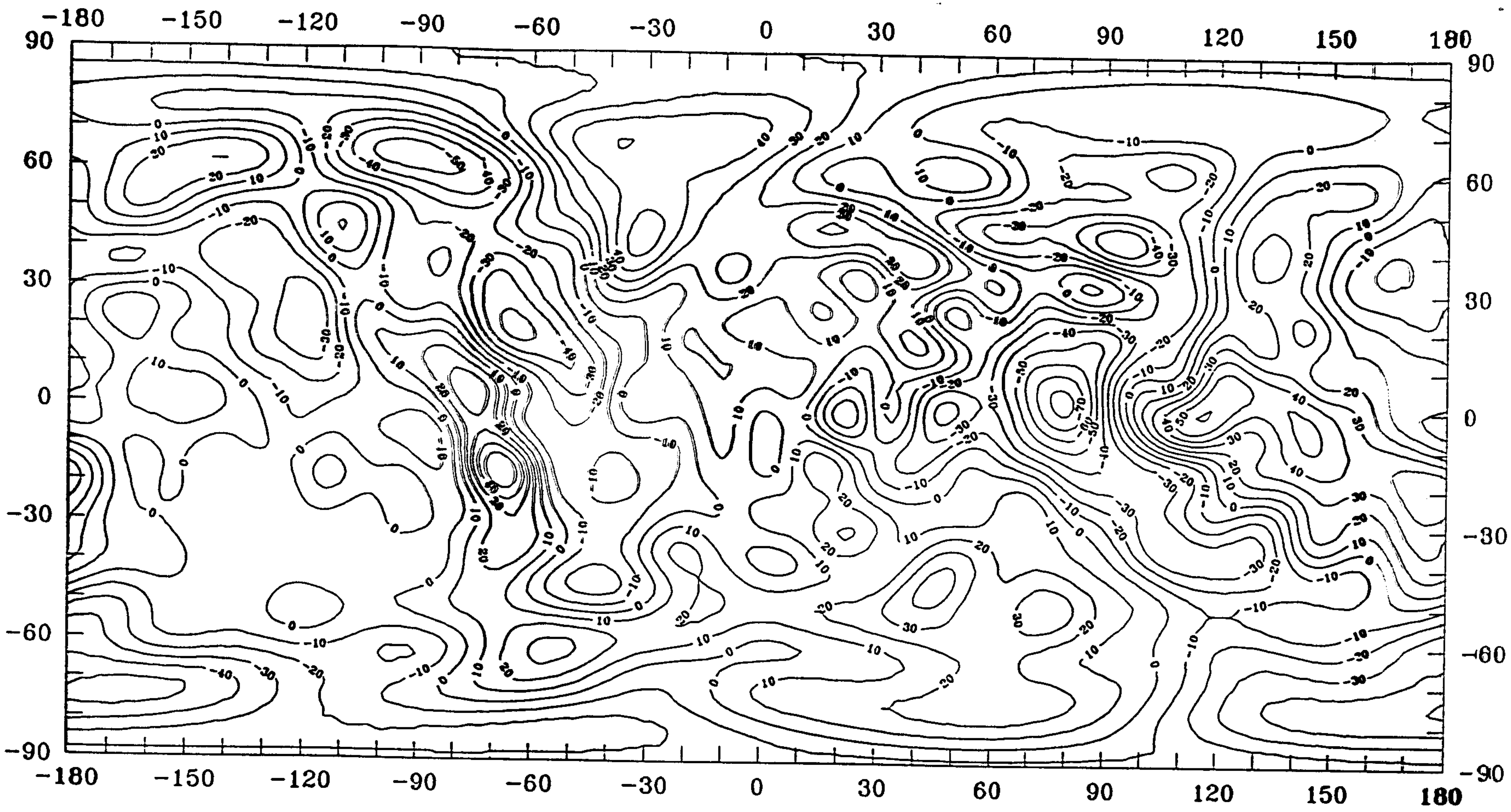
(7 + 100 + 1000 Point Masses): PM1107/01

Input Data: **OSU 89B up to (60,60) minus PM107/01**

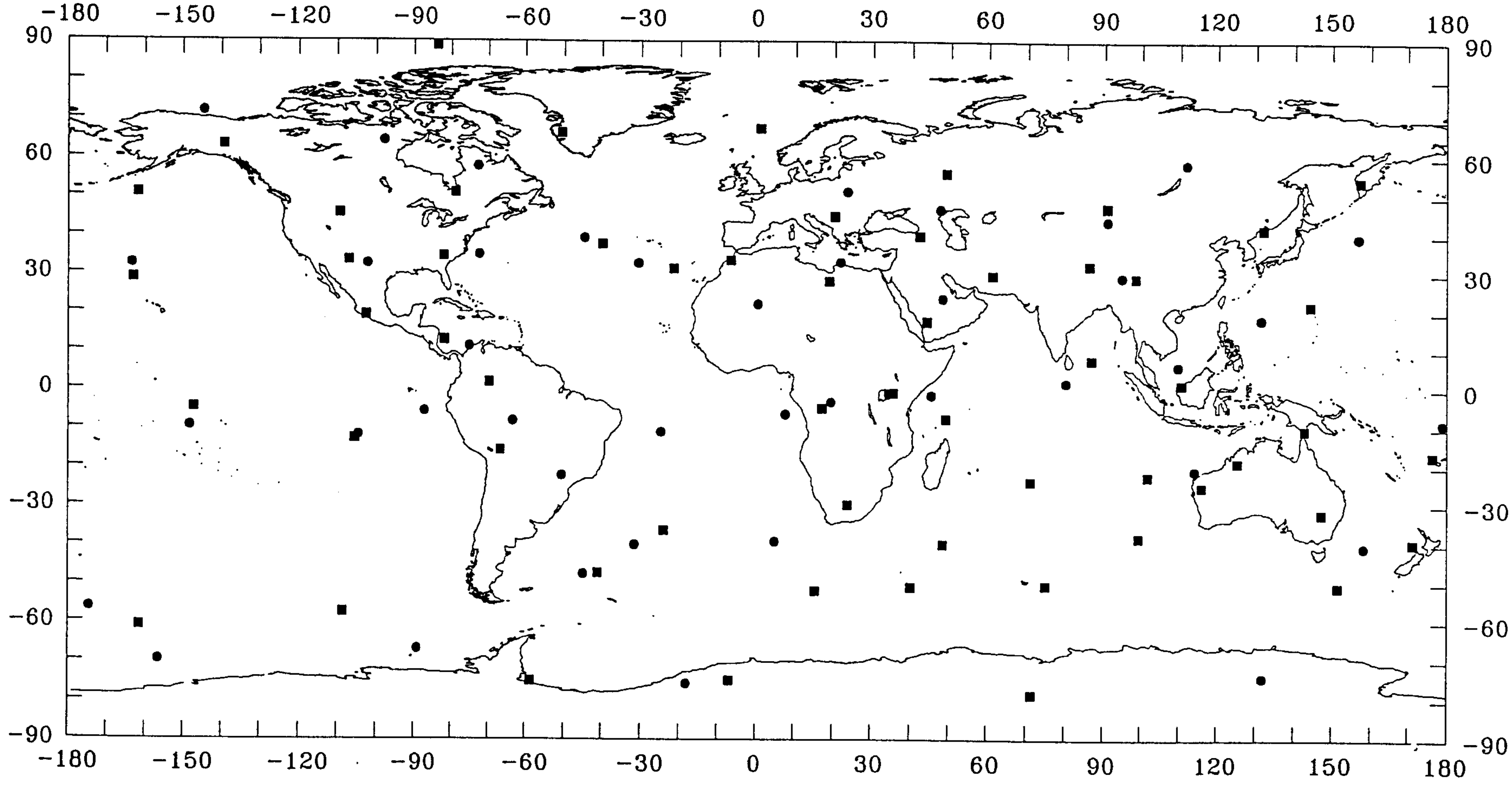
- **for position optimization:
  gridded acceleration vectors on a reference sphere**

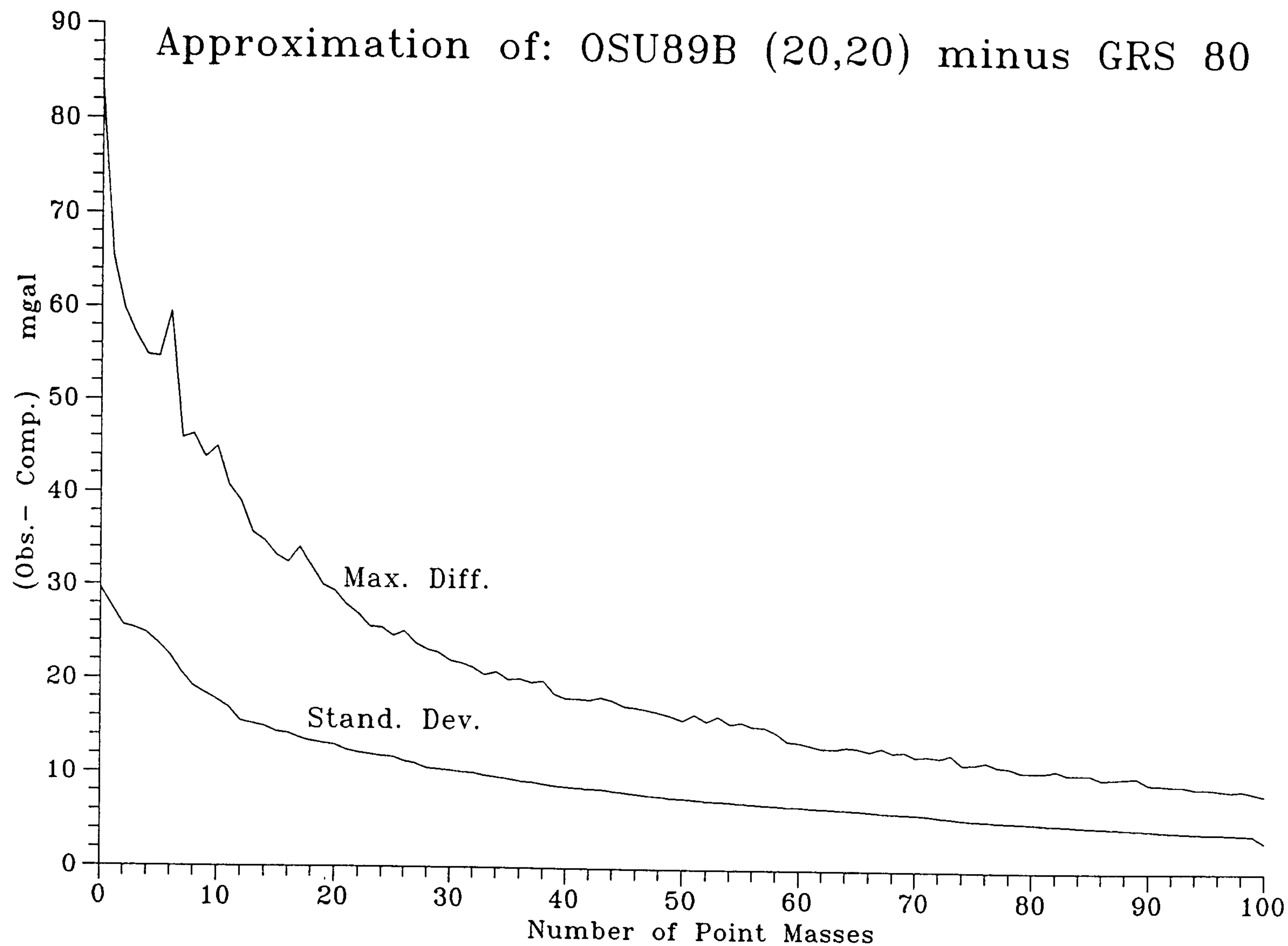
- **for final adjustment of masses (positions fixed):
  spherical harmonic coefficients, weighted with respect to their variance and their influence at 500 km altitude**

Gravity Disturbances OSU89B (20,20) (Isol. 10 mgal)

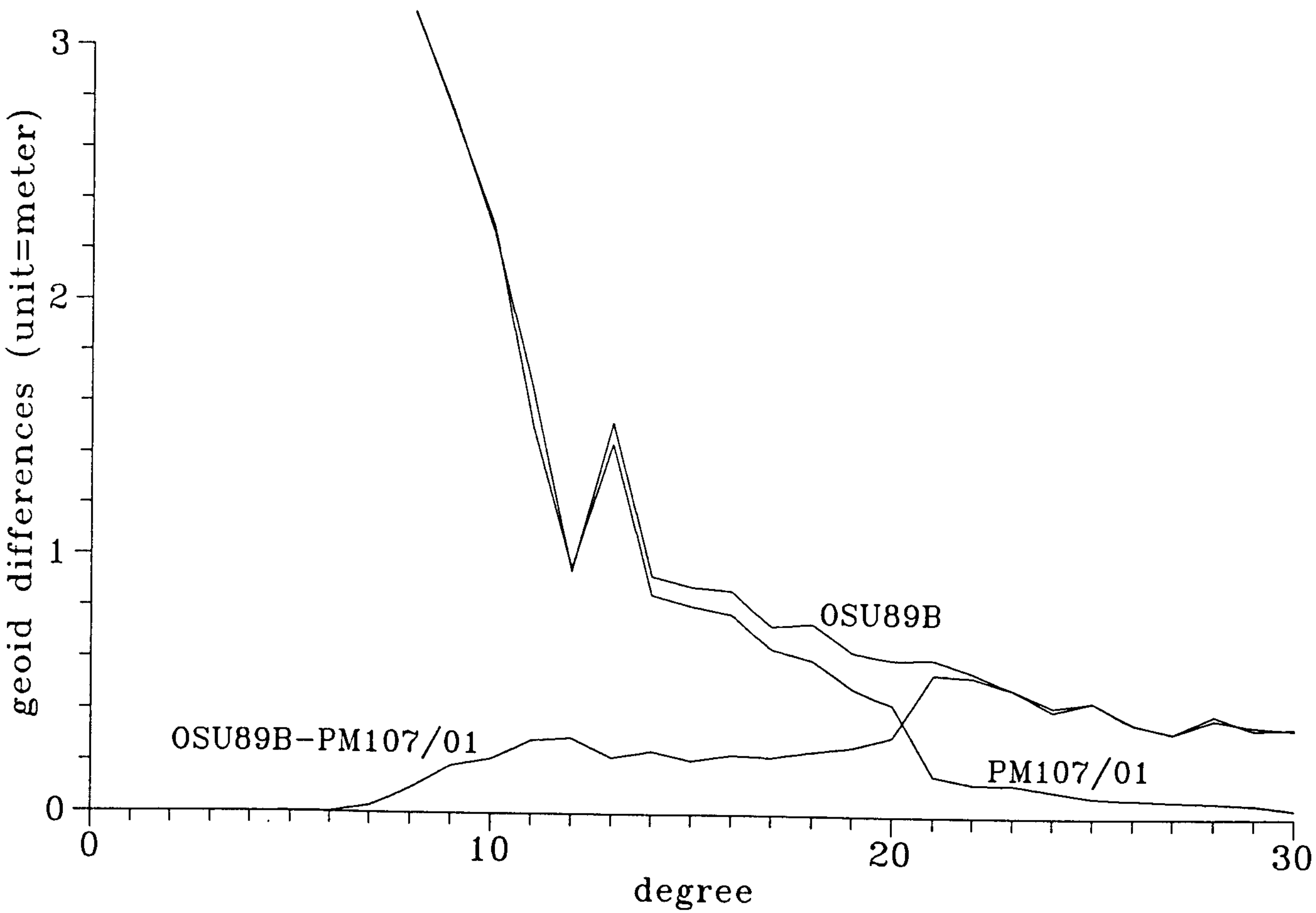


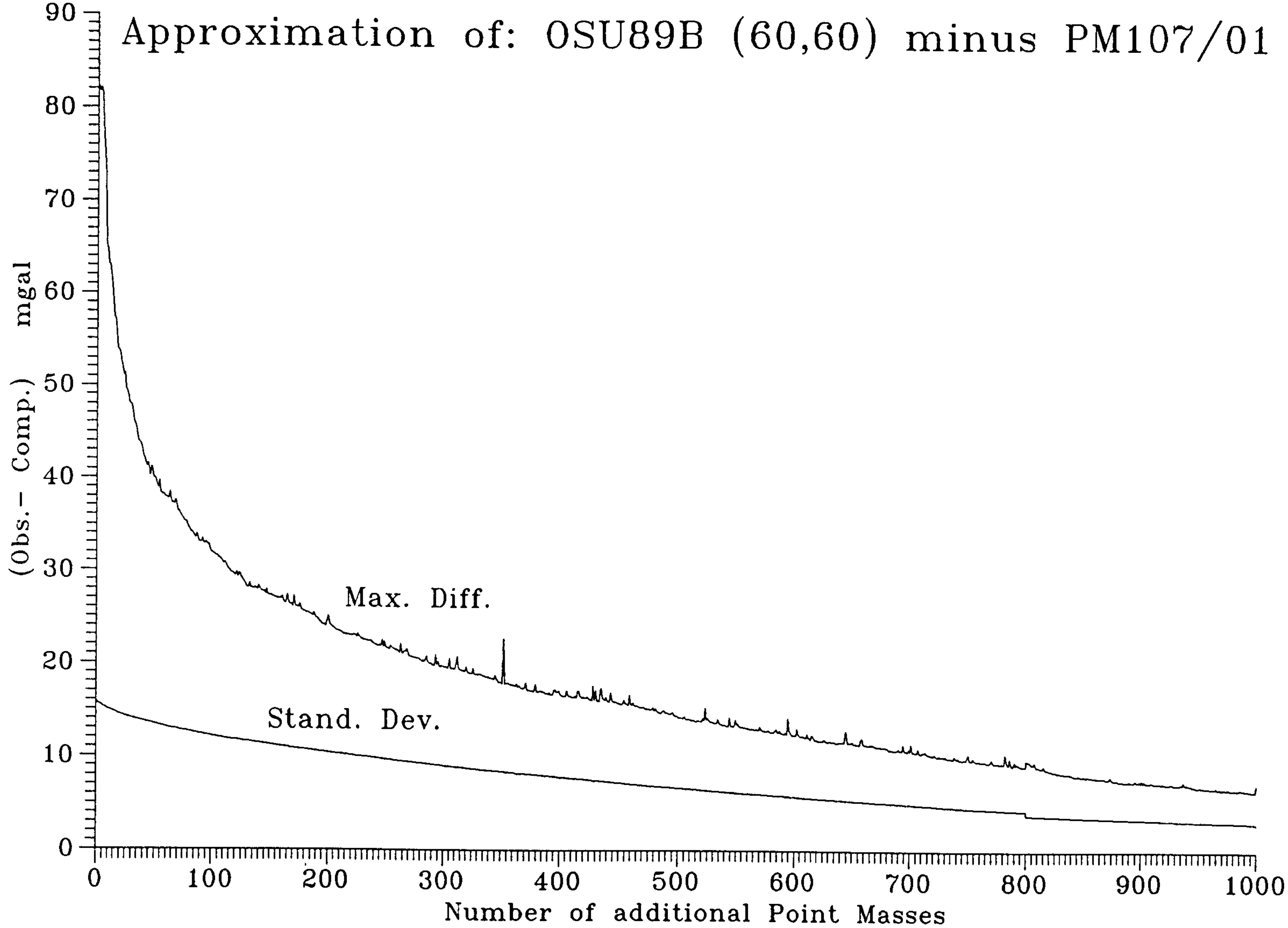
100 PM approx. OSU89B (20,20) minus GRS 80 • – negativ ■ – positiv



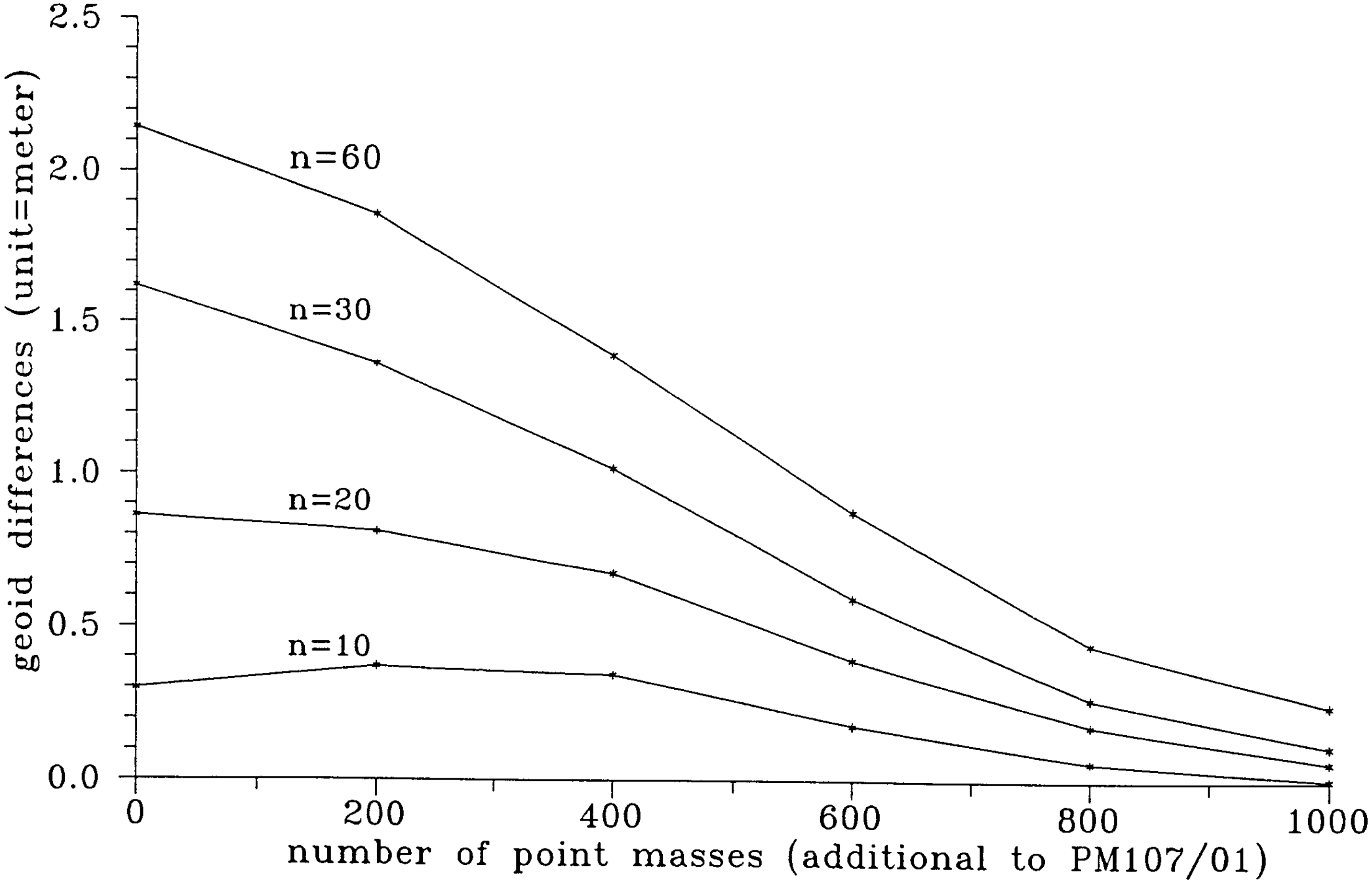

Approximation of: OSU89B (20,20) minus GRS 80
(Obs.- Comp.) mgal
Number of Point Masses
Max. Diff.
Stand. Dev.
0
10
20
30
40
50
60
70
80
90
100

Geoid undulations of OSU89B and PM107/01
and their differences (by spherical harmonic degree)

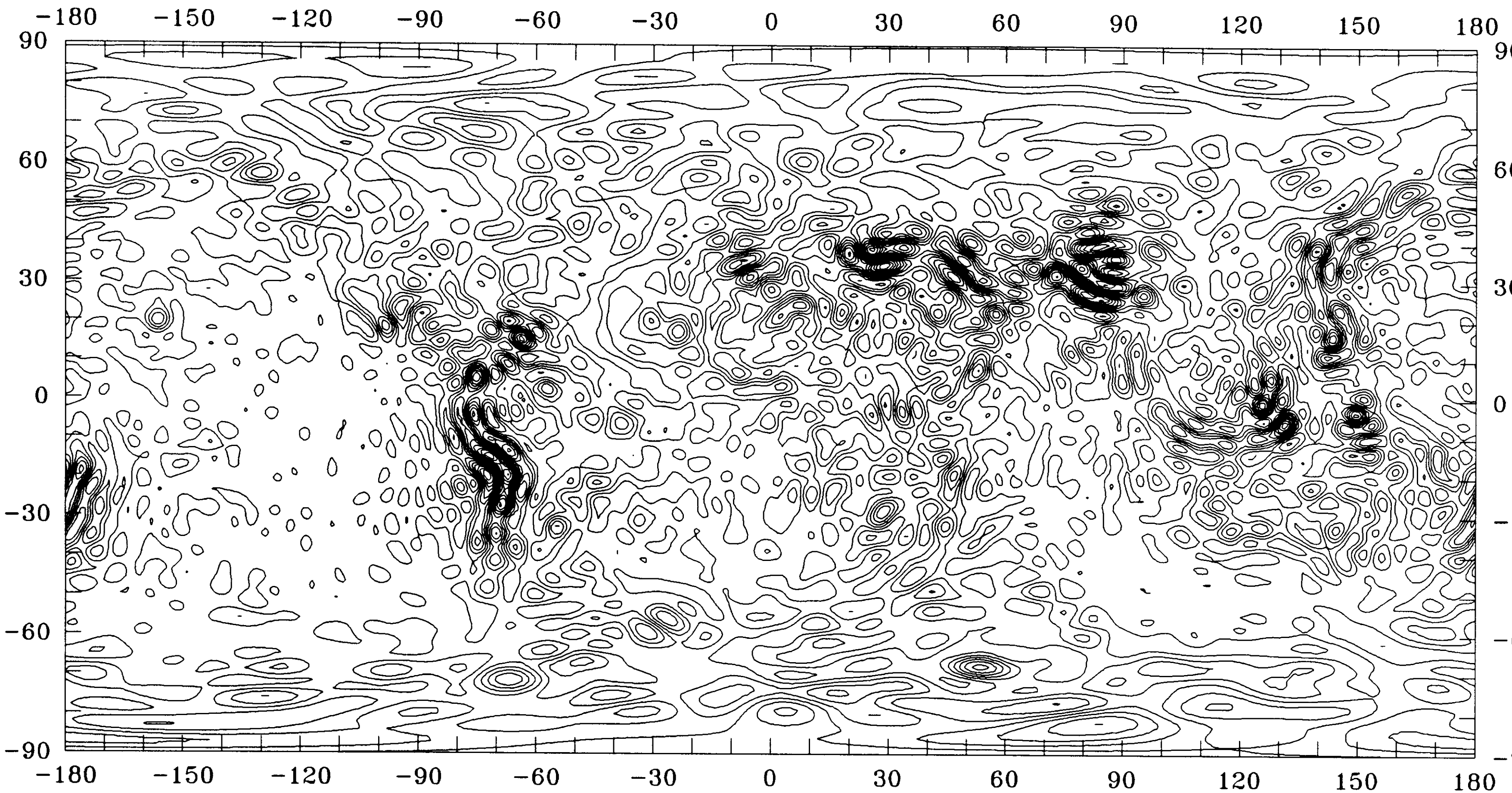



Approximation of: OSU89B (60,60) minus PM107/01
(Obs.- Comp.) mgal
Max. Diff.
Stand. Dev.
Number of additional Point Masses
0
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000

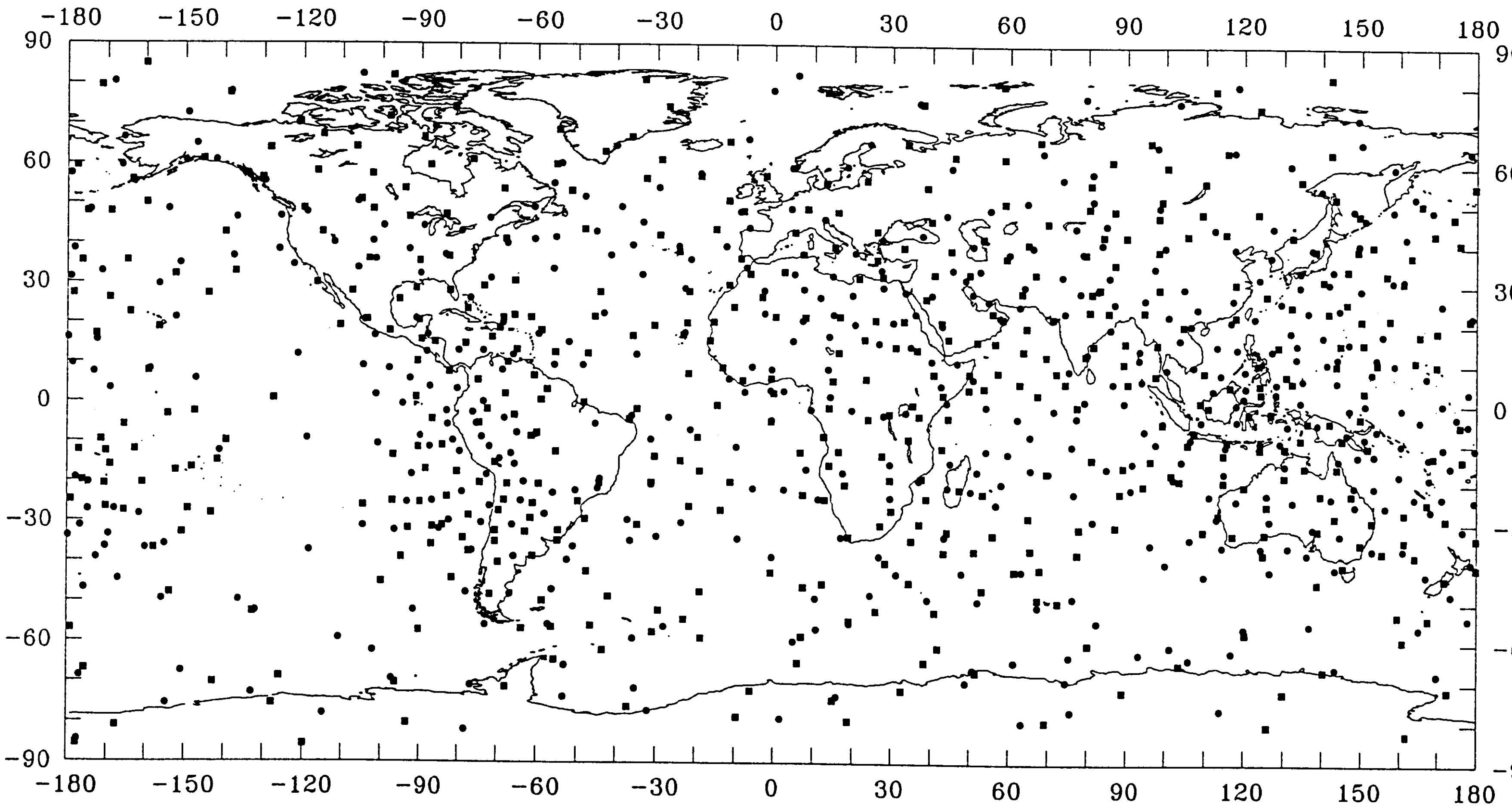
Geoid differences between OSU89B (up to degree n) and the corresponding spectral part of the (increasing number of) point masses



Grav. Disturb. OSU89B (60,60) minus PM107/01 (Isol. 10 mgal)



1000 PM approx. OSU89B (60,60) minus PM107/01 • – negativ ▪ – positiv



5.4 Regional Model for Europe

Region: $-20° < \lambda < +50°$
$+30° < \phi < +75°$

$(\underbrace{7 + 100 + 1000}_{\text{global}} + 1000 \text{ Point Masses})$

Aim:

Methodological investigation for approximation of the gravitational field (target: geoid computation) with OSU89B-resolution based on point masses (e.g. as a reference model for local geoid determination)

Input Data:

OSU89B up to (360,360) minus global model PM1107/01

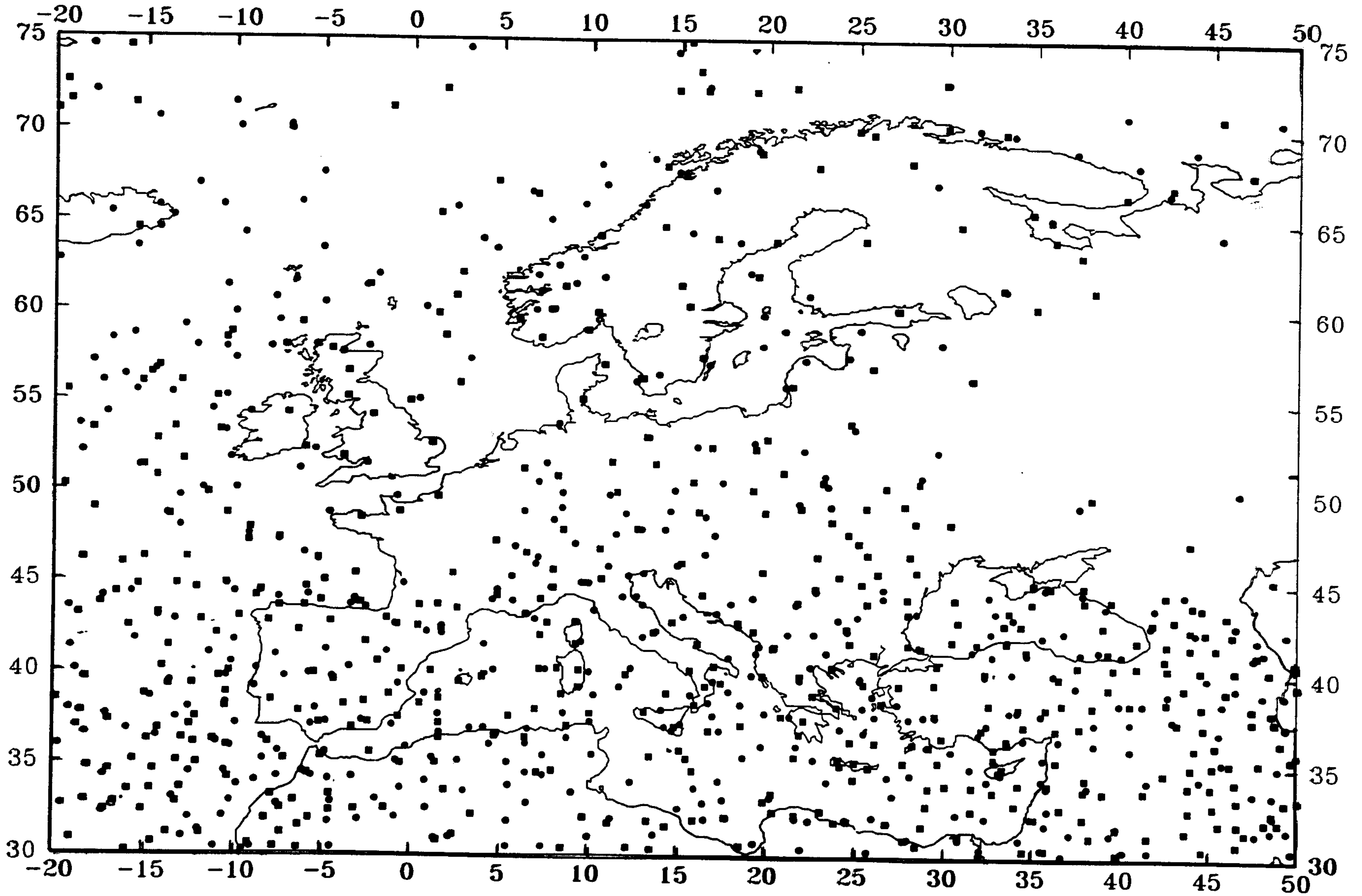
- for position optimization:
  gridded acceleration vectors on a reference sphere

- for final adjustment of masses (positions fixed):
  gridded acceleration vectors on a reference sphere plus 5 geoid heights (for better absolute orientation of geoid surface)

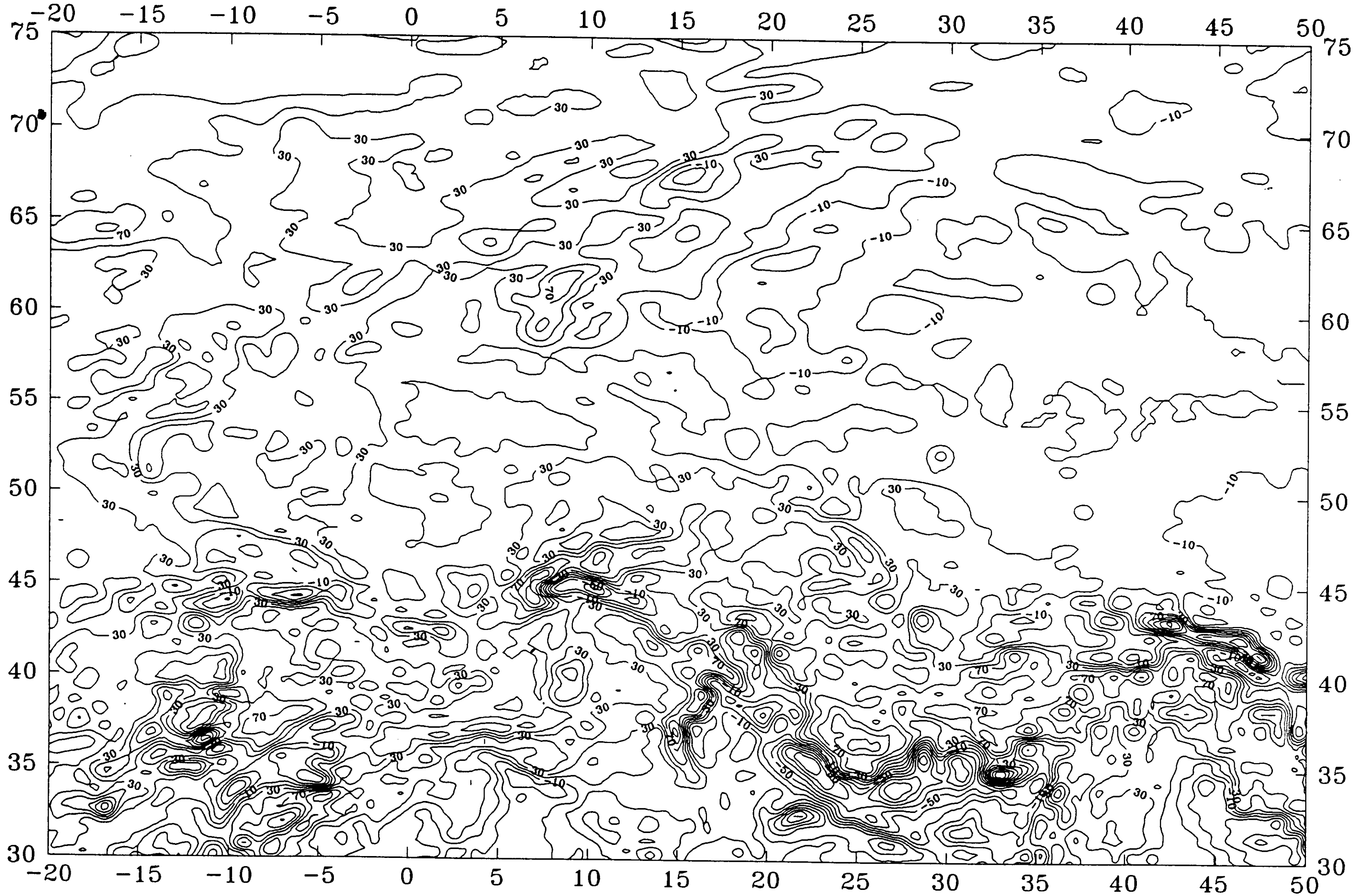
Evaluation:

- fit of the model to the input data
- fit of the model to 1000 randomly choosen control points (gravity disturbances and geoid heights → "true" error) and comparison to accuracy prediction using linear error propagation (formula (12))

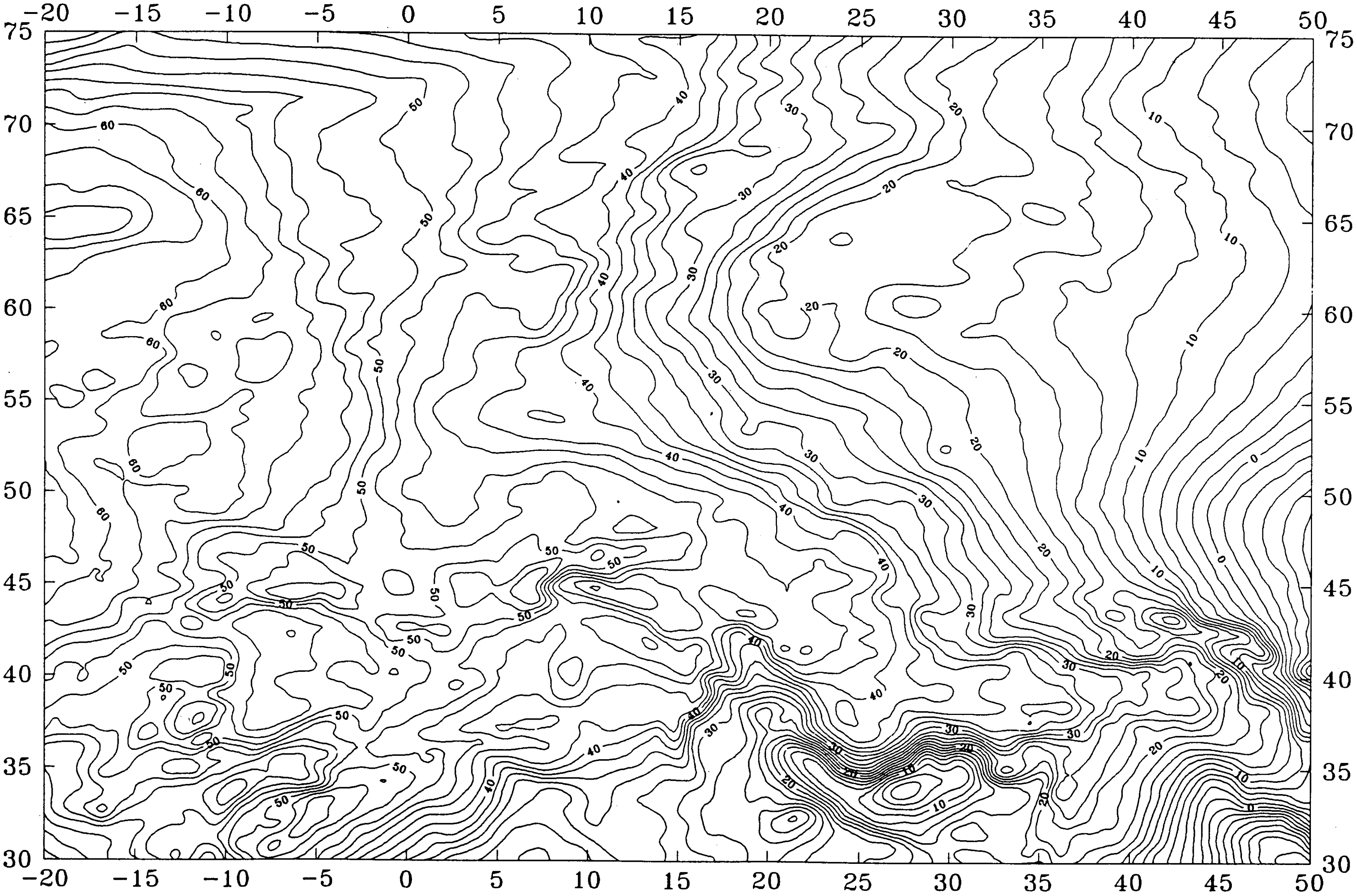
1000 Point Masses approx. OSU89B minus PM1107/01 for Europe



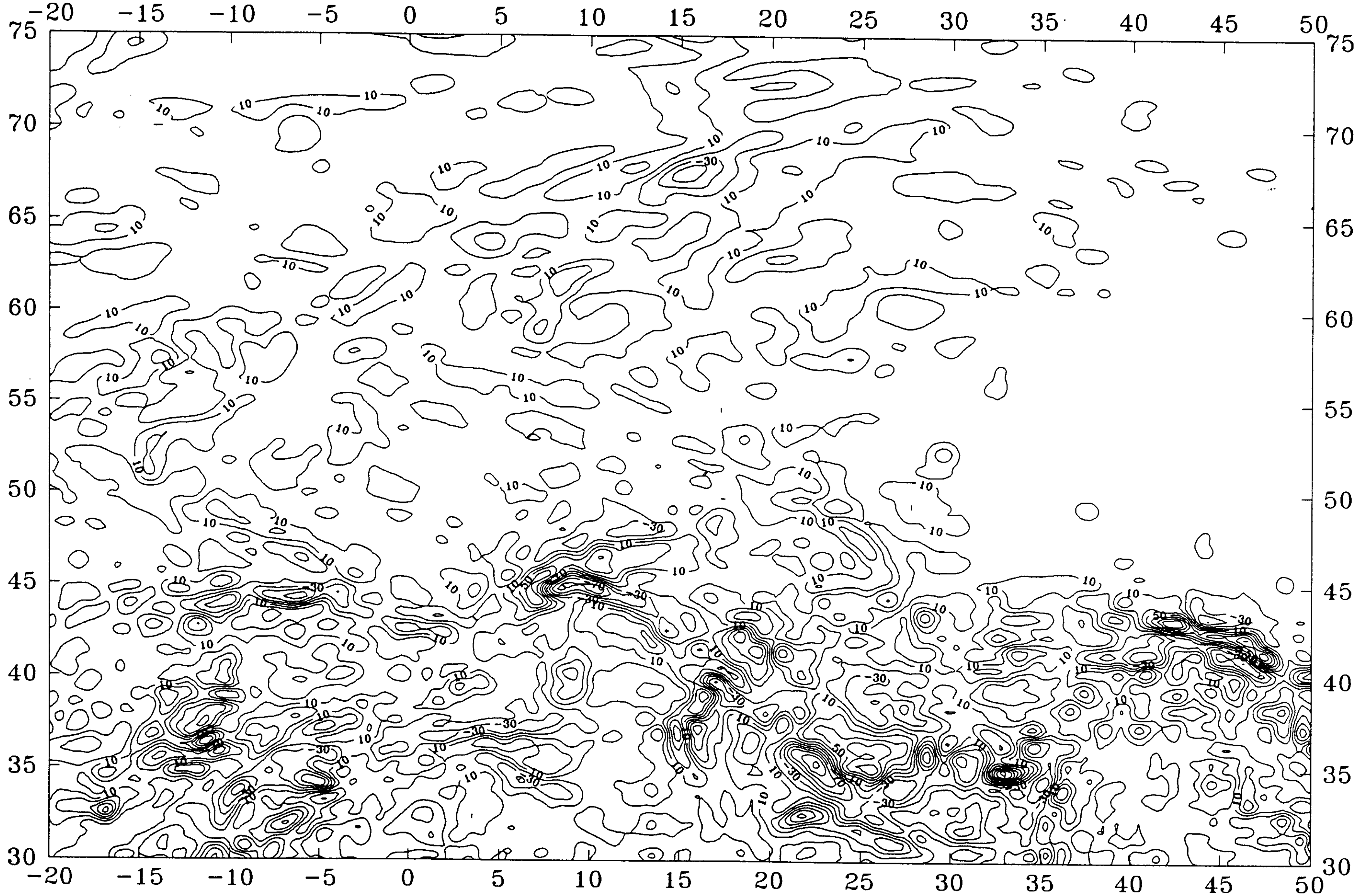
Grav. Disturb. OSU89B minus GRS80 for Europe (Isol. 20 mgal)



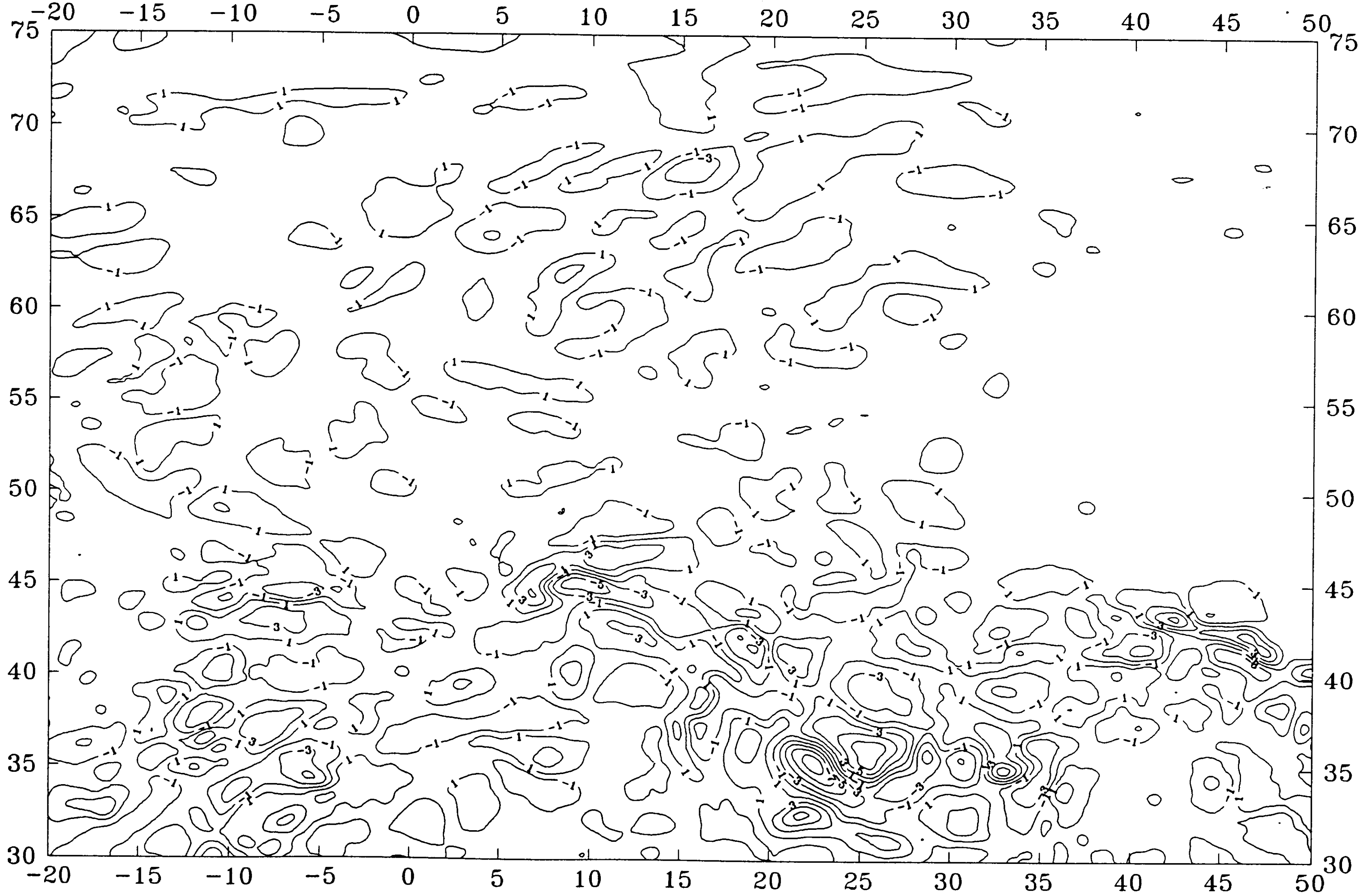
Geoid OSU89B minus GRS80 for Europe (Isol. 2.00 m)



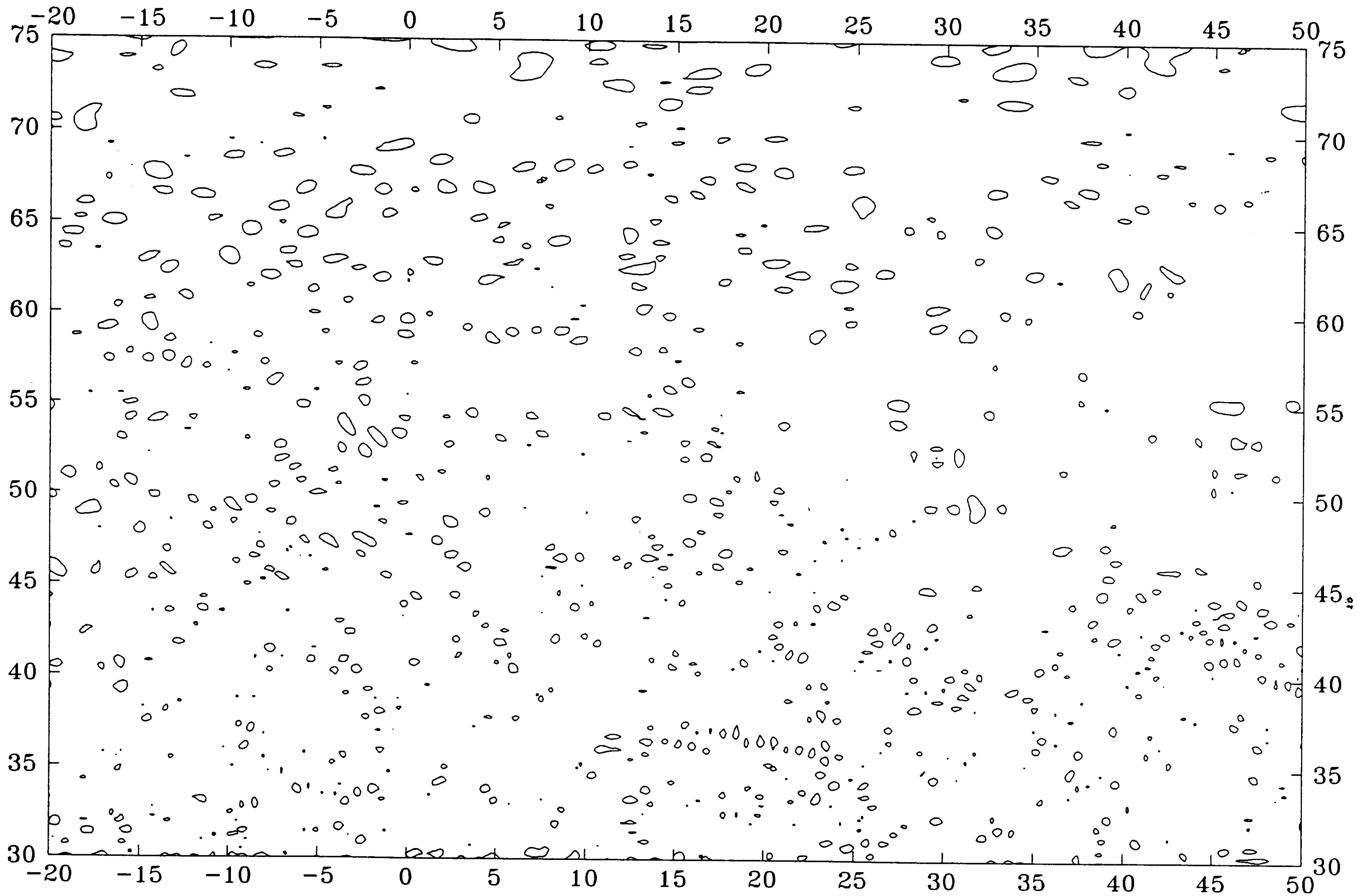
Grav. Disturb. OSU89B minus PM1107/01 for Europe (Isol. 20 mgal)



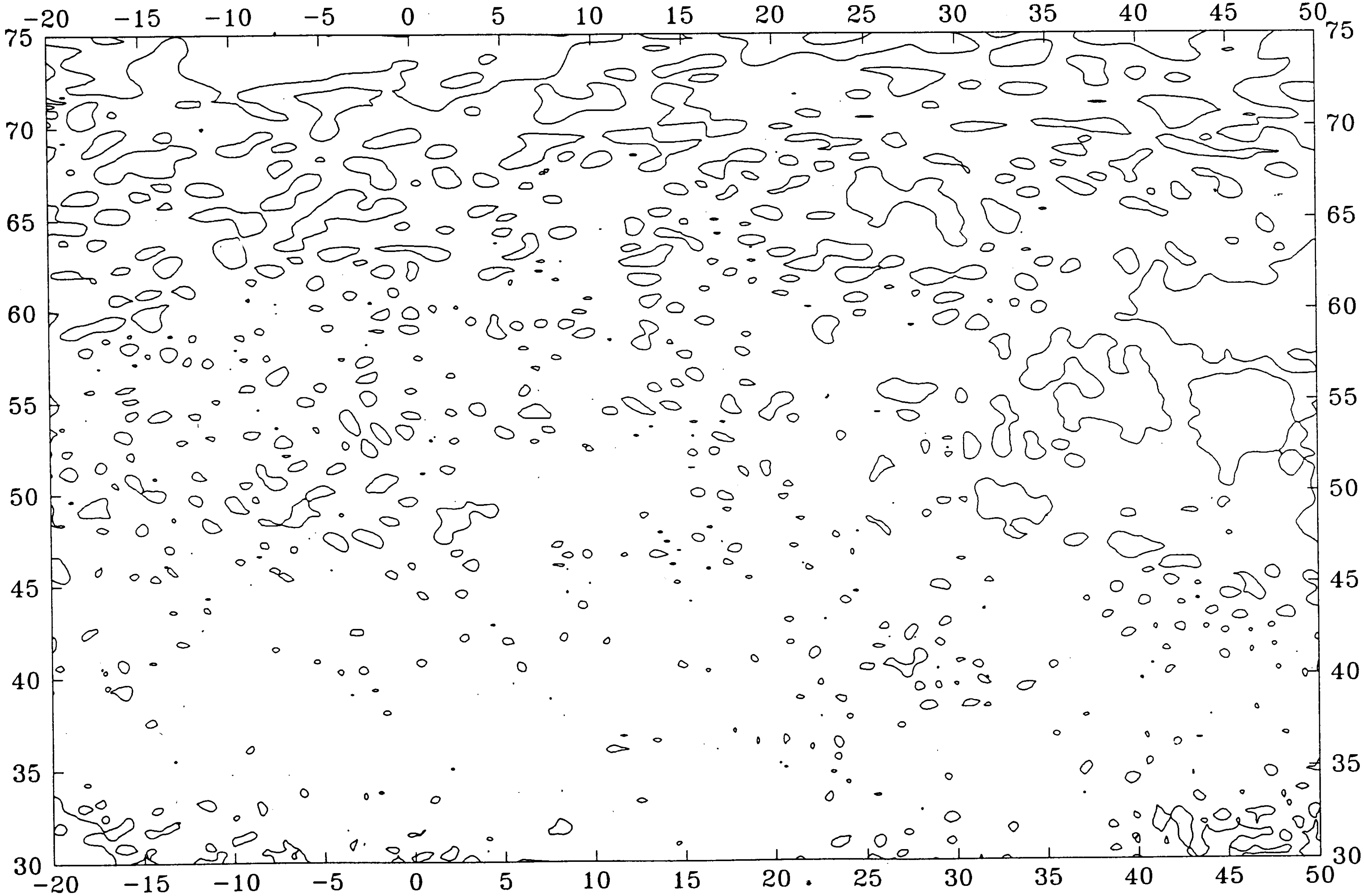
Geoid OSU89B minus PM1107/01 for Europe (Isol. 2.00 m)

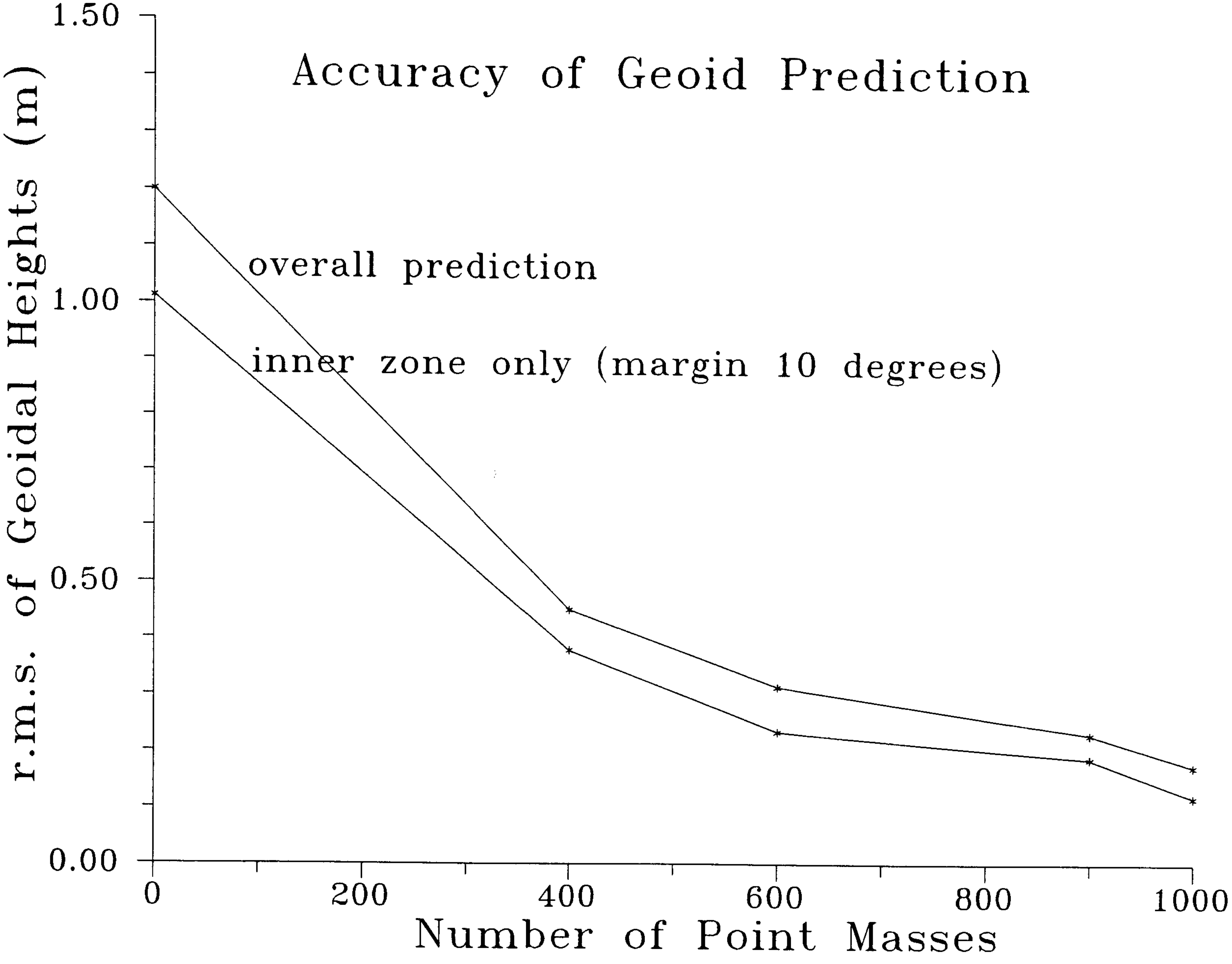


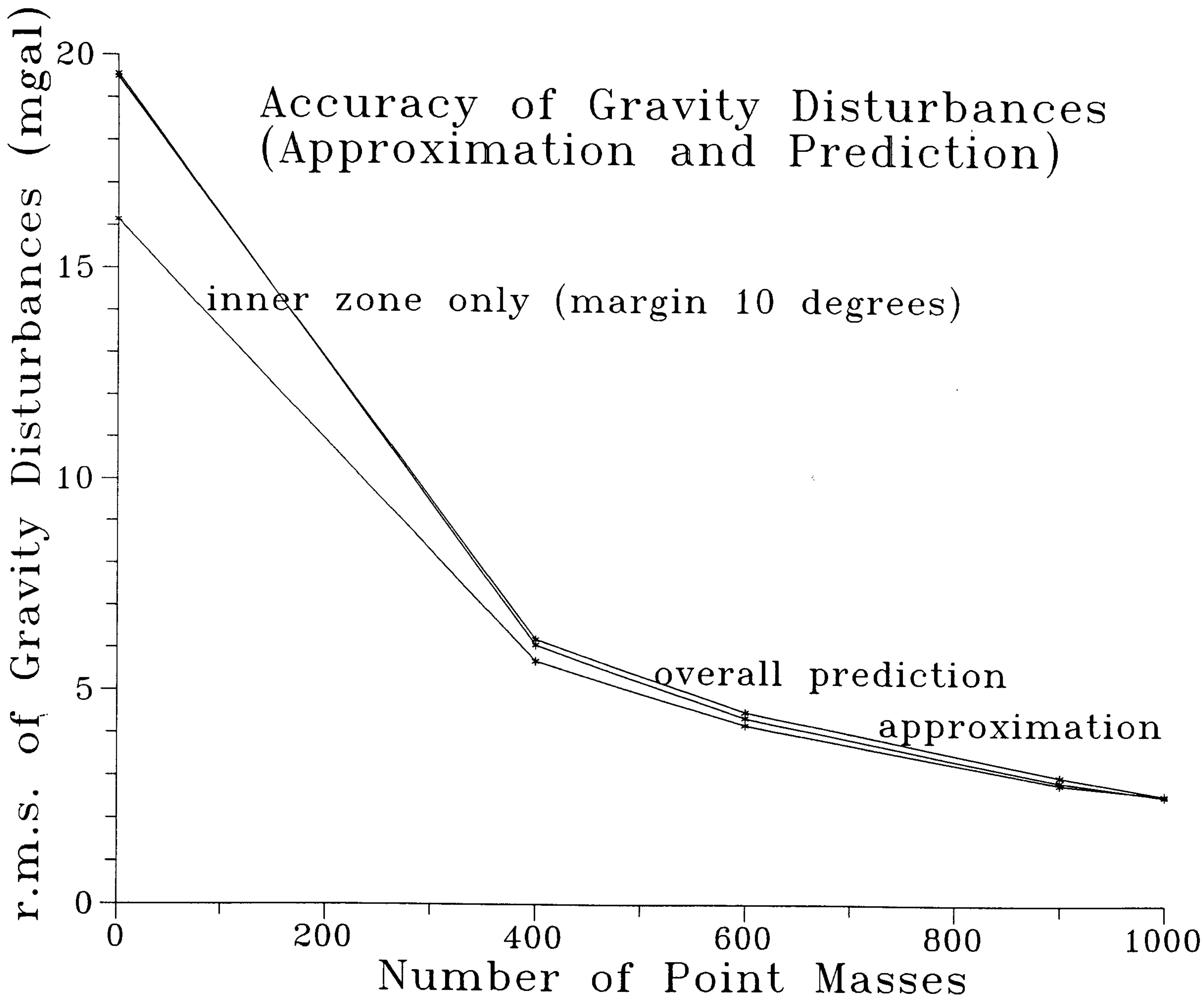
Grav. Disturb. OSU89B minus PM2107/01 for Europe (Isol. +/-5 mgal)



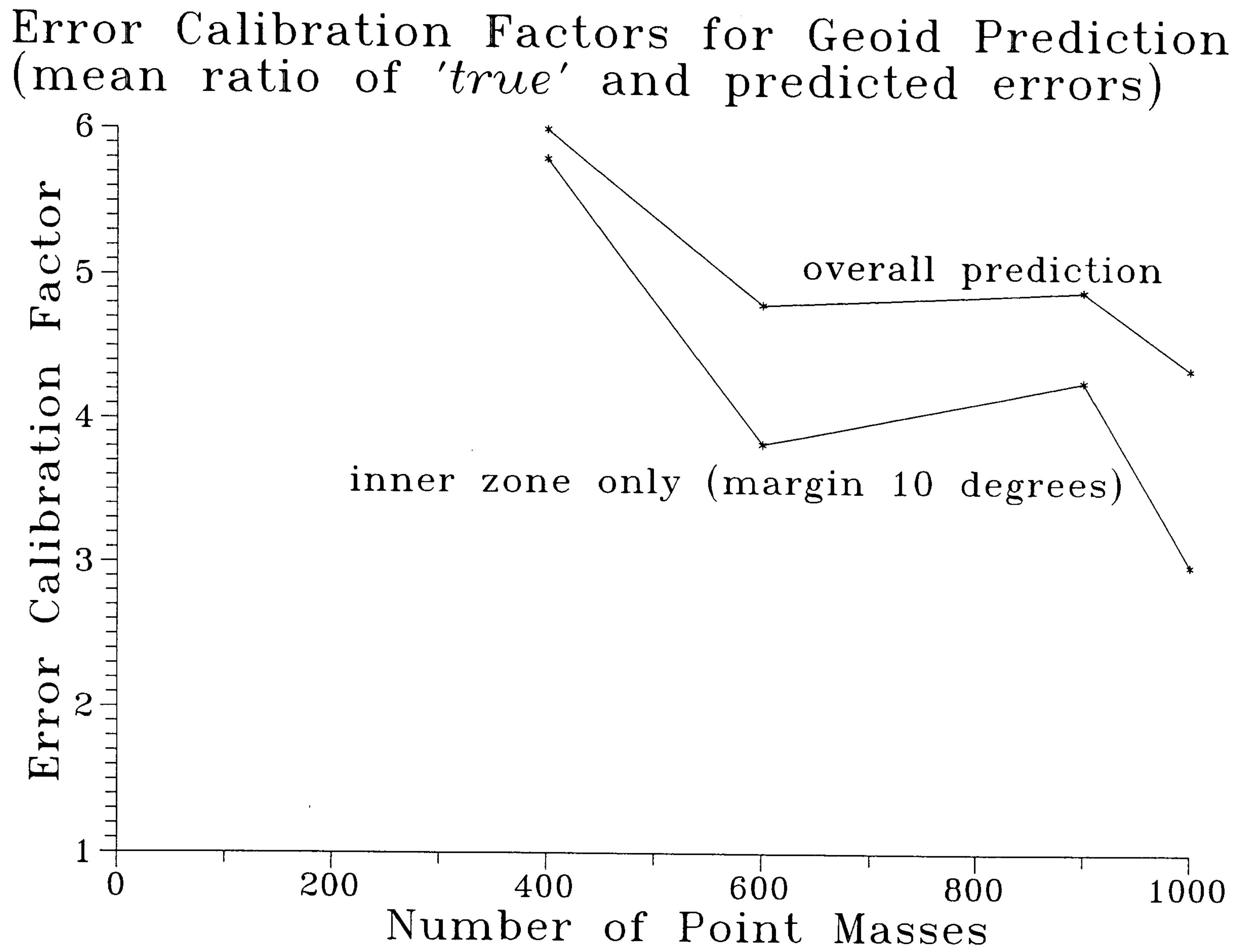
Geoid OSU89B minus PM2107/01 for Europe (Isol. +/−0.20 m)




Accuracy of Geoid Prediction
overall prediction
inner zone only (margin 10 degrees)
r.m.s. of Geoidal Heights (m)
1.50
1.00
0.50
0.00
0
200
400
600
800
1000
Number of Point Masses



Accuracy of Gravity Disturbances
(Approximation and Prediction)
inner zone only (margin 10 degrees)
overall prediction
approximation
r.m.s. of Gravity Disturbances (mgal)
Number of Point Masses
0
5
10
15
20
0
200
400
600
800
1000



Error Calibration Factors for Geoid Prediction
(mean ratio of 'true' and predicted errors)
6
5
4
3
2
1
Error Calibration Factor
overall prediction
inner zone only (margin 10 degrees)
0
200
400
600
800
1000
Number of Point Masses

5.5. Global Model with High Resolution

(7 + 100 + ... Point Masses)

Input Data: **OSU 89B up to (360,360) minus PM107/01**

- gridded acceleration vectors on a reference sphere

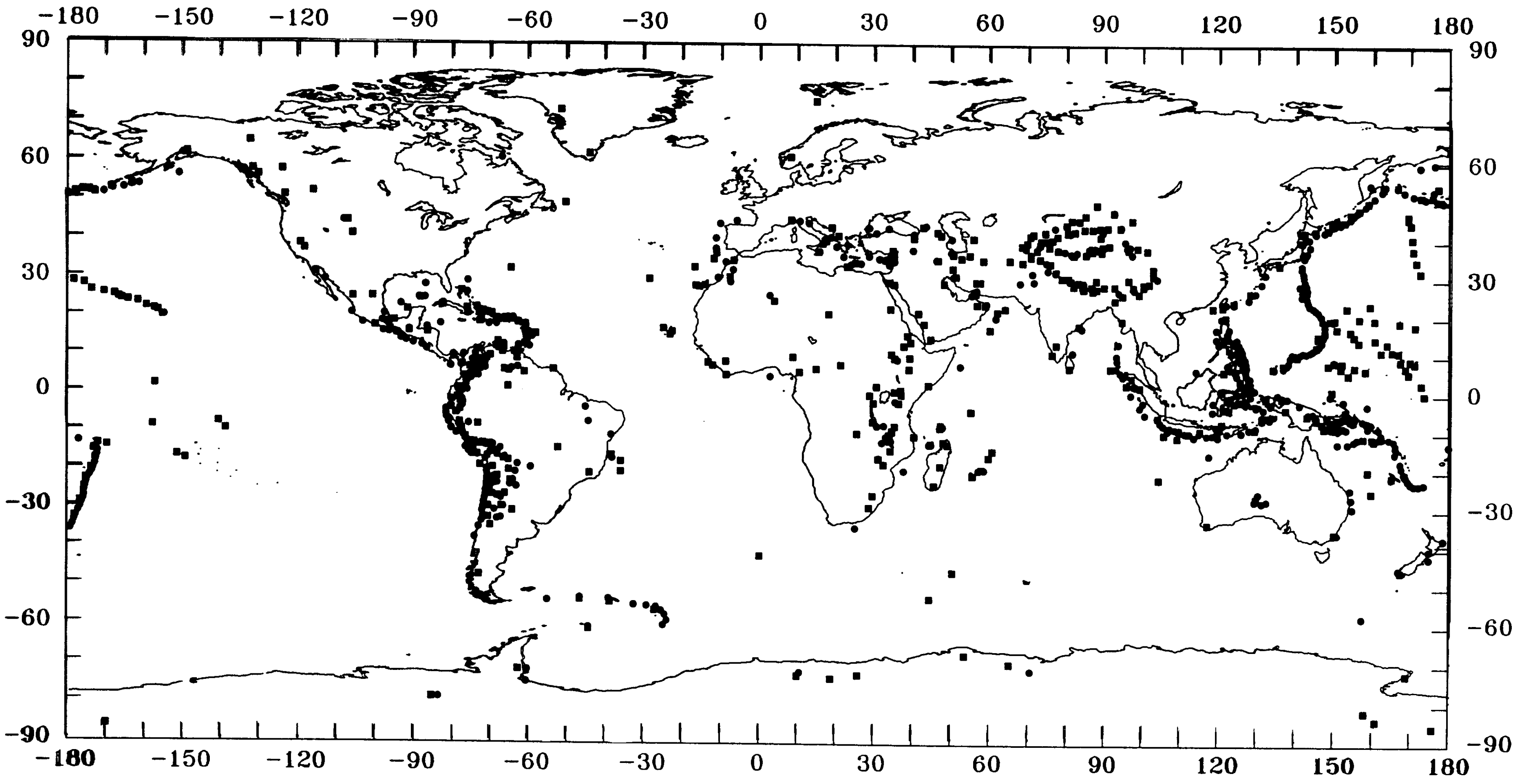
Computational Results:

(The stepwise approximation is not yet finished)

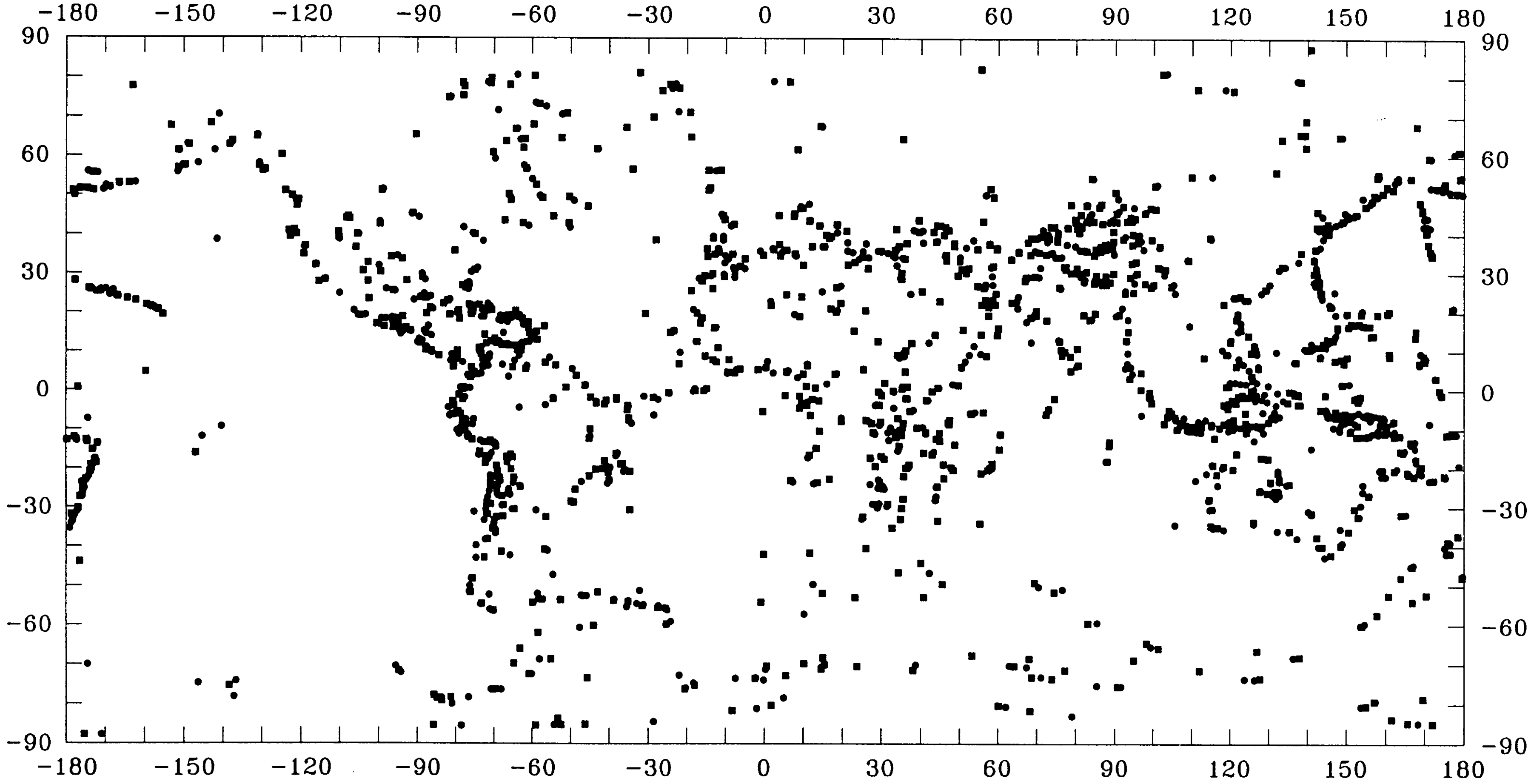
Positions of the first 1000 and 4000 Masses are presented in the figures

→ close relations to plate boundaries !

1000 PM approximating OSU89B (360,360) minus PM107/01



4000 PM approximating OSU89B (360,360) minus PM107/01



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